MATHEMATICS SPECIALIST
UNIT 3
Section One:
Calculator-free

Semester 1, 2016

Student Number: In Figures

Student Number: In Words

Time allowed for this section
Reading time before commencing work: five minutes
Working time for this section: fifty minutes

Materials required/recommended for this section
To be provided by the supervisor
This Question/Answer Booklet
Formula Sheet

To be provided by the candidate
Standard items: pens, pencils, pencil sharpener, eraser, correction fluid/tape, ruler, highlighters

Special items: nil

Important note to candidates

No other items may be used in this section of the examination. It is **your** responsibility to ensure that you do not have any unauthorised notes or other items of a non-personal nature in the examination room. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of exam
Section One: Calculator-free	7	7	50	53	35
Section Two: Calculator-assumed	13	13	100	97	65
			Total	150	100

Instructions to candidates

1. The rules for the conduct of Western Australian external examinations are detailed in the *Year 12 Information Handbook 2016*. Sitting this examination implies that you agree to abide by these rules.

2. Write your answers in this Question/Answer Booklet.

3. You must be careful to confine your response to the specific question asked and to follow any instructions that are specified to a particular question.

4. Spare pages are included at the end of this booklet. They can be used for planning your responses and/or as additional space if required to continue an answer.
    *   Planning: If you use the spare pages for planning, indicate this clearly at the top of the page.
    *   Continuing an answer: If you need to use the space to continue an answer, indicate in the original answer space where the answer is continued, i.e. give the page number. Fill in the number of the question that you are continuing to answer at the top of the page.

5. **Show all your working clearly**. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.

6. It is recommended that you **do not use pencil**, except in diagrams.

7. The Formula Sheet is **not** to be handed in with your Question/Answer Booklet.

Section One: Calculator-free **35% (53 Marks)**

This section has **seven (7)** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time for this section is 50 minutes.

Question 1 **(5 marks)**

Use the fact that the polynomial $P(x) = x^5 + x^3 - 2x$ is divisible by $D(x) = x^2 - 1$, to find all real and complex numbers that satisfy the equation:

$$x^5 - x = x - x^3$$

(5 marks)

Question 2 **(6 marks)**

A sphere has equation $x^2 + y^2 + z^2 - 2x + 4y + 3z + 1 = 0$.

(a) Determine the coordinates of the centre and the radius of the sphere. (3 marks)

(b) Determine the vector equation of the straight line that passes through the points on the sphere where $y = -2$ and $z = 0$. (3 marks)

See next page

Question 3   **(7 marks)**
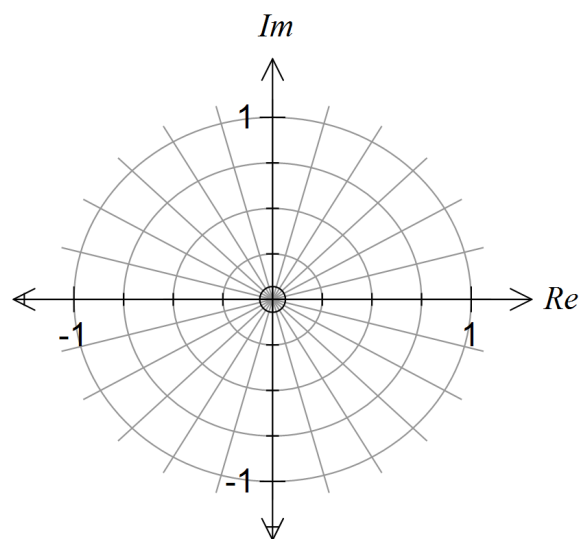
(a)   Let $z = 2\cos\left(\dfrac{2\pi}{3}\right) + 2i\sin\left(\dfrac{2\pi}{3}\right)$.

   (i)   Express z in Cartesian form.   (2 marks)

   (ii)   Determine z^5 in Cartesian form.   (3 marks)

(b)   If $w^3 + 1 = 0$, sketch the location of all roots of this equation on the axes below.

                                                                                                       (2 marks)

Question 4   **(15 marks)**

(a)   Determine the vector equation of the plane that contains the points A(1, -1, 2), B(2, 1, 0) and C(3, -1, 1).   (4 marks)

(b)   Plane Π has equation $x + 2y - z = 3$. Line L is perpendicular to Π and passes through the point (1, -6, 4). Determine where line L intersects plane Π.   (4 marks)

See next page

(c)   The shortest distance between the point Q with position vector $\langle 4,3,k \rangle$ and the plane $\boldsymbol{r} \cdot \langle 1,2,2 \rangle = 3$ is 3. Determine the value of k.

(4 marks)

(d)   Determine distance between two parallel planes Π_1 and Π_2.
        Where Π_1: $r \cdot \langle -1,2,-2 \rangle = 9$ and Π_2: $r \cdot \langle 2,-4,4 \rangle = 24$.

(3 marks)

See next page

Question 5   **(7 marks)**

Consider the following system of equations, where k is a real constant.

$$x + 2y + z = 3$$
$$2x - y - 3z = k$$
$$x + 3y + kz = 6$$

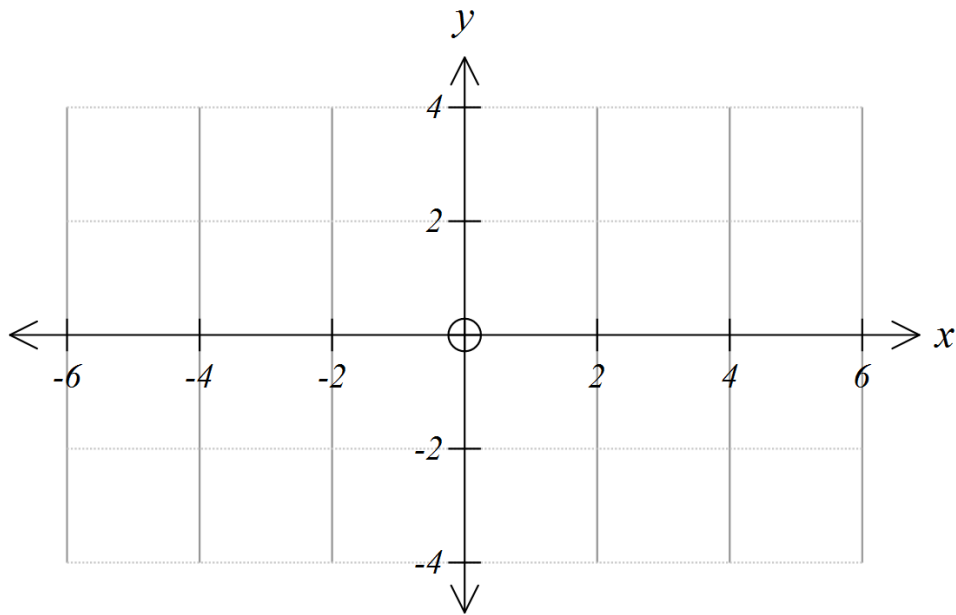(a)   Solve the system of equations when $k = 1$.   (3 marks)

(b)   Show that no value of k exists for the system of equations to represent three planes intersecting in a single straight line.   (4 marks)

Question 6   **(7 marks)**

(a)   Sketch the graph of $y = \dfrac{|x-2|}{2}$ on the axes below.   (2 marks)

(b)   Solve the equation $4|x-8| = 38 - x$.   (3 marks)

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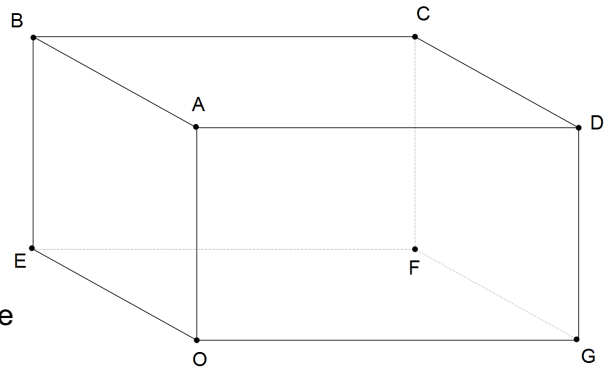(c)   Solve the inequality $\dfrac{1}{|x+2|} \leq 1$.   (2 marks)

Question 7 **(6 marks)**

In the rectangular prism shown, $|\overrightarrow{AD}| = 4$ $|\overrightarrow{AB}| = 3$ and $|\overrightarrow{BE}| = 2$.

Let P be the midpoint of BC, and Q be a point on FG such that FQ : QG = 1 : 2.

Let $\overrightarrow{OG}$, $\overrightarrow{OE}$ and $\overrightarrow{OA}$ represent the x, y and z axes respectively, with *i, j* and *k* as their respective unit vectors.

(a) Express each of the following in terms of *i, j* and *k*.

(i) $\overrightarrow{EC}$ (1 mark)

(ii) $\overrightarrow{PG}$ (1 mark)

(iii) $\overrightarrow{QA}$ (1 mark)

(b) Determine the Cartesian equation of the sphere that contains all the vertices of this prism.

(3 marks)

End of questions

Additional working space

Question number: _____

Additional working space

Question number: _____

Additional working space

Question number: _____